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LEVITATION OF DENSE MATERIALS USING A  
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# Stabilized Acoustic Levitation of Dense Materials Using a High-Powered Siren

Paul M. Gammell  
Arvid Croonquist  
Taylor G. Wang



December 15, 1982

**NASA**

National Aeronautics and  
Space Administration

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## ABSTRACT

Stabilized acoustic levitation and manipulation of dense (e.g. steel) objects of 1 cm diameter, using a high-powered siren, was demonstrated in trials that investigated the harmonic content and spatial distribution of the acoustic field, as well as the effect of sample position and reflector geometries on the acoustic field. Although further optimization is possible, the most stable operation achieved is expected to be adequate for most containerless processing applications. Best stability was obtained with an open reflector system, using a flat lower reflector and a slightly concave upper one. Operation slightly below resonance enhances stability as this minimizes the second harmonic, which is suspected of being a particularly destabilizing influence.



## I. INTRODUCTION

Acoustic levitation has been proposed as a means of containerless processing of materials. Previous investigators have used a variety of devices to achieve acoustic levitation; the continuing problem has been to provide power levels strong enough to levitate dense objects and, at the same time, to design a system that can provide stabilized levitation of the objects.

For example, stable acoustic levitation against the earth's gravity in a gaseous medium has been demonstrated by Wang et al. (1974) and others (Whymark et al., 1979).

These studies were limited to low density samples because the acoustic power levels produced in the resonant chambers by conventional electro-acoustic transducers (loudspeaker coils and piezoelectric devices) were only 140 to 150 dB. These sources, which have the advantage of being well defined as to geometry and performance, can be operated over a wide frequency range.

More recently, Lee and Feng (1982) have demonstrated the stable levitation against the earth's gravity and manipulation of high density submillimeter objects using a focussing radiator to produce sound levels of 172 dB by concentrating the energy from individual 130 PZT transducers.

St. Clair (1941) produced high intensities for the acoustic levitation of heavy objects by using a magnetic driver and a resonant cylinder. This device makes use of the high-Q resonance of an aluminium cylinder and, in the form he used it, could not be operated over a range of frequencies as the driver itself is resonant and there appears to be no easy way to change its resonance frequency to compensate for the change of the acoustic resonance of the region between the reflectors. (The resonant frequency of the region between the reflectors is affected by temperature, humidity, and sample size.)

Allen and Rudnick (1947) demonstrated the levitation of high density objects of a few centimeters dimension by using a high powered siren. A properly designed siren can produce extremely high intensities, with an efficiency of over 50% (Jones, 1946). The siren is particularly efficient when its high acoustical impedance is matched to the impedance of the air by an exponential horn (Jones, 1946; Allen and Rudnick, 1947). Furthermore, the frequency of the siren can be readily varied, which is not the case with the St. Clair resonant device.

Although a device based on the siren to levitate heavy objects of 1-2 cm diameter was successfully demonstrated by Allen and Rudnick, constant adjustments must be made to the siren frequency to maintain the object in the acoustic field. Even with the most careful manual manipulation of the siren frequency, the sample occasionally underwent large oscillations for no apparent reasons and eventually ejected from the field. Objects could not be levitated for long periods of time, as is necessary for containerless processing, without close operator attention.

## II. THE SIREN AND ASSOCIATED SYSTEMS

The goal of the present study was to identify the causes of the instability experienced in the study by Allen and Rudnick and to remedy the instability so that the siren may become the basis of an acoustic levitation facility suitable for containerless processing of high density objects of over 1 mm size in the earth's gravity. Specifically, we wished to find a reflecting geometry that would give better stability than the parallel reflectors previously used. We also wished to identify the factors that affect the strength and stability of acoustic levitation.

A siren produces sound by periodically interrupting an otherwise steady flow of compressed air. Typically, this is done by forcing the compressed air through two carefully aligned rows of holes in two closely spaced metal plates, one of which is stationary and the other of which rotates. The rate at which these holes fall momentarily into alignment becomes the fundamental frequency of the resulting highly nonsinusoidal waveform.

In this conventional implementation the siren is a ring source. The resulting spatial pressure distribution can be used to advantage when levitating objects in an intrinsically cylindrical reflector geometry.

Sirens have several fundamental drawbacks. One is that the acoustic pressure waveform is rich in harmonics, which would be expected to affect the stability with which a sample is levitated and positioned because multiple modes are excited in the resonant reflector system, producing conflicting force distributions. The pressure and temperature of the air supplied to the siren by the compressor is subject to variations which affect the sound pressure level of the siren output and the resonant frequency of the reflector system, respectively. The pressure varies because compressors are mechanical devices that are typically designed for heavy-duty rather than precision applications. The air is heated by compression, and its temperature when it reaches the siren depends on the thermal losses in the interconnecting plumbing which, in turn, depend on such factors as the length of time the system has been running, the ambient temperature, ventilation, etc. The siren speed, and thereby the acoustic frequency, is determined by the speed of the motor. Since this motor is of the order of a horsepower, it would normally be of the induction type, which, of necessity, operates with a certain amount of "slip" with respect to the line frequency (Puchstein et al., 1960).

Our siren design is similar to the one used by Allen and Rudnick (1947). The rotor and stator each have 100 holes of 2.54 mm diameter on a 76.4 mm radius. The rotor is mounted on a precision bearing that allows adjustment of the clearance between the rotor and stator and is driven by a 1.5 hp variable-speed motor. The acoustic impedance of the siren is matched to the air by a tapered horn section designed to approximate the exponential horn of Rudnick (1974). The siren was mounted with the axis of rotation vertical. The acoustic reflectors were symmetric about this axis. The sound diffracted around the lower reflector into the space between the reflectors. This design is expected to minimize the influence of the supply air flow on the levitated sample and its positional stability.

Fig. 1 shows the relation of the siren, the horn, the collimating section following the horn, and the reflector system used to produce the standing waves required for acoustic levitation.

In our system the drift of the motor speed was reduced by a servo system to maintain constant voltage to the motor. This produced adequate stability of the motor speed to maintain the sample levitated in a stable position for over an hour. The day-to-day reproducibility of the siren frequency was about  $\pm 10$  Hz for a given setting of the control. In practice the operator would determine the precise frequency to use by observation of the behavior of the sample. The main reasons that the siren had to be re-tuned frequently was the slow drift of the motor speed (which determines the siren frequency) and the variation of the temperature of the air (which determines the resonant frequency). Improved stability, if needed, could have been provided by an outer feedback loop, in order to servo the set point of the motor voltage controller to keep the frequency (or motor speed) constant.

A first order correction to the drift of the temperature of the air supply was provided by a heat exchanger to remove the compressor heat from the air supplied to the siren. This prevented the upward drift of the resonance frequency as the plumbing from the compressor heated up. Although this improvement was adequate, additional stability could be provided by regulating the temperature of the air supply.

The air supply system was also equipped with a moisture trap. Elimination or control of the moisture content of the air is important as it is another factor which affects the resonance frequency of the reflector system.

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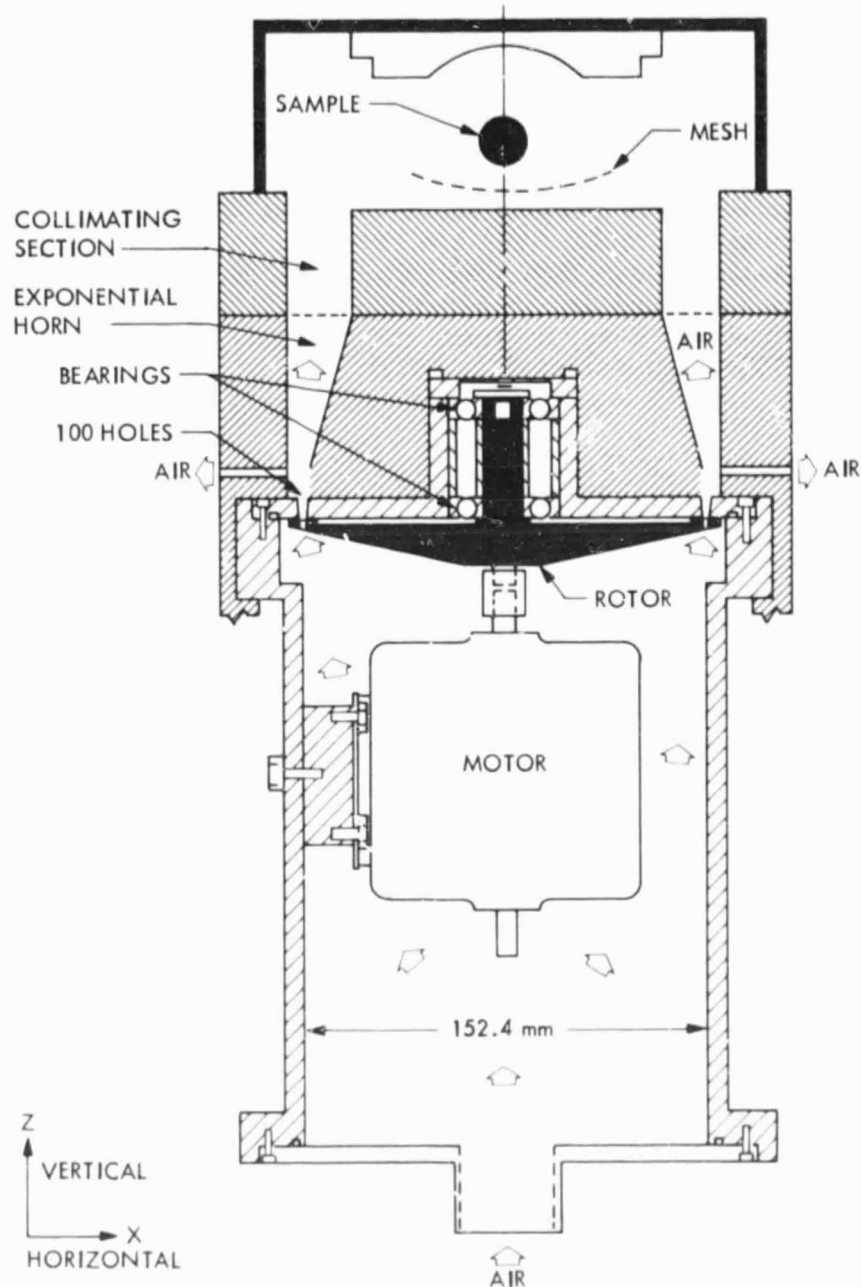


Figure 1. Cross section of siren, motor, and reflector system

The possibility of other improvements was also identified, such as ballasting or control of the pressure of the air supply. These changes were not implemented as they were not found to be crucial at this stage of development.

A sample can be positioned in the acoustic field by radiation pressure alone. This requires that standing waves be present which, in turn, requires some sort of reflector system. The sample will be driven towards the minimum of the ambient (time-averaged) pressure field. For the case where the density of the sample is much greater than that of the medium, the radiation pressure is described by the Bernoulli equation (Wang, Saffren, and Elleman, 1977; Wang, Saffren, and Elleman, 1974; Landau and Lifshitz, 1959; and Westervelt, 1977):

$$\langle P \rangle = (\overline{p^2}/2\rho c^2) - (1/2)\rho(\overline{V^2})$$

where

$\langle P \rangle$  is the radiation pressure  
 $p$  is the excess acoustic pressure  
 $V$  is the gas particle velocity  
 $\rho$  is the density of the gas  
 $c$  is the velocity of sound in the gas  
 and a bar over a quantity denotes a time average.

Clearly, the ambient pressure will be a minimum at the velocity antinodes, where  $v^2$  is a maximum and  $p^2$  is a minimum (ideally zero). Thus, a levitated sample will be forced towards the velocity antinodes. Since the pressure gradient, and therefore the force at the reflector surface is zero, the sample must initially be supported away from the surface, as by the mesh shown in Fig. 1.

Out of consideration for the safety of the operator and the disposition of the other occupants of the building, the siren levitation system was enclosed in a special enclosure, fitted with a glass observation window. An electrically driven traverse was used to expedite field mapping. Even with the acoustic insulation, the sound levels were about 75 dB at 1 meter from the observation window, and the operator used ear protection consisting of both ear plugs (rated at 40 dB at 3 kHz) and ear phones (rated at over 24 dB at 3 kHz). Double ear protection was used to provide an extra margin of safety and it was not assumed that the dB protection would necessarily be additive.

### III. MEASUREMENT TECHNIQUES

#### A. Microphone and Associated Electronics

The sound pressure was measured using Kistler Type 6001 quartz pressure transducers, which are capable of withstanding the highest sound pressure encountered. The output was amplified by a Kistler 5004 charge amplifier. The voltage from this amplifier was converted to a decibel reading by a Hewlett-Packard model 3403C digital voltmeter. The manufacturer's calibration of the transducer was used to relate the electrical charge output to the incident acoustic pressure. The data is reported as a SPL (sound pressure level) in dB re  $2.0 \times 10^{-4}$  dyne  $\text{cm}^{-2}$ . The noise level of the microphone and amplifier is 130 dB, which is low enough for these high intensity applications.

The output of this amplifier, which is an accurate representation of the sound pressure waveform, was recorded on an oscilloscope when waveforms were reported and was analyzed with a Panoramic LP-1A sweeping spectrum analyzer (20 Hz to 20 KHz) for harmonic analysis. The accuracy of the spectrum analyzer is about  $\pm 1$  dB.

For precise measurements the frequency was measured from this signal using a Hewlett-Packard 5243L counter. When data was acquired more rapidly, the calibration of the motor speed control, which was previously checked against the counter, was used, reproducing the frequency to an accuracy of  $\pm 10$  Hz.

The acoustical output of the siren is reasonably constant even if no effort is made to automatically regulate the pressure head.

When studying the factors that affect the acoustic output, choice of the microphone location is important for accurate measurements of the acoustic field, since the microphone should not perturb the field appreciably while measuring it faithfully. For measuring the frequency dependence, this was solved by making the microphone (Kistler type 6001, 6.3 mm diameter) an integral part of and flush with the face of the lower reflector. Since the microphone was located in the center of the reflector, this has the additional advantage of being at the location of the pressure maximum for all modes of the lowest radial order.

When the sound field was to be mapped, a microphone small enough not to seriously perturb the sound field was needed, together with a positioning apparatus. The same type of Kistler

6.3 mm diameter microphone was supported by a 6.3 mm rod mounted axially, so that the rod and microphone together had the same acoustic profile as a single 6.3 mm diameter rod. Although this presents a slight perturbation to the sound field, the data is nevertheless felt to be representative. The microphone was traversed horizontally (perpendicular to the axis of the siren) across the sound field by a leadscrew mechanism that stepped the microphone at precisely 1.81 mm increments.

Throughout this text "vertical" will refer to the direction of the axis of the siren, in units that start at  $z=0$  at the surface of the lower reflector and increase toward the upper reflector, as shown in Fig. 1. The horizontal direction is perpendicular to this axis of symmetry of the siren and the reflectors. When the absolute sound pressure level is plotted against the microphone position, the horizontal distance is expressed in units which are normalized by the size of the lower reflector: The distance of the microphone from the center of the reflectors is divided by the radius of the lower reflector. This number is zero when the microphone is in the middle and  $\pm 1$  when the microphone is at the edge of the lower reflector. Since the field is expected to be azimuthally symmetric, the field was measured on only one side of the center of the reflector. To provide a check on the alignment of the traverse apparatus and the symmetry of the field, data was taken over a region which extended to 25.4 mm the other side of the middle of the reflector region. Traverses were made with the microphone at varying heights above the lower reflector, which was a parameter of these families of plots.

#### B. Restrained Samples

Some experiments involved the effect of the sample position. Measurements of the sound pressure in the presence of an unrestrained levitated sample are complicated by the wandering of the sample and by the fact that it would be attracted to the microphone by the same acoustic forces that cause two or more levitated objects to be drawn together. To overcome this problem a restrained sample was used. This consisted of a 12.7 mm diameter steel ball that was attached to a thin rod. This restrained sample was positioned in the field at the elevation a sample would normally levitate and at the horizontal position either in the center of the reflector, or to one side, as dictated by the criteria of the experiment. When the sound field was mapped with the restrained sample in place at a chosen location, the 6.3 mm diameter microphone was traversed horizontally at a designated height.



#### IV. MEASUREMENTS AND OBSERVATIONS

##### A. Frequency Response and Pressure Field of Siren

1. Siren Alone. The acoustical output of the siren is reasonably constant over the frequency range of interest for these levitation studies. Figure 2 shows the frequency dependence of the pressure measured at the center of a flat reflector located atop the rotor housing, with the upper reflector removed, over a frequency range around that used for levitation. The microphone was an integral part of the reflector. Since the diameter of the reflector is only of the order of a wavelength, the sound is expected to be diffracted around it and into the levitation region. The acoustical output of the siren is fairly constant over the frequency range of interest, showing a trend that decreases only 4.5 dB from 2900 Hz to 3800 Hz.

On top of this constant level there was a fine variation of approximately  $\pm 1$  dB with a periodicity of 140 Hz. When the acoustical pressure waveform was examined carefully on an oscilloscope, it was seen to be amplitude modulated by 10% at around 140 Hz. This is undoubtedly due either to slow fluctuations of the compressor output, resonated by the plumbing, or to the effect of the resonance of the plumbing on the impedance of the air supply as seen by the siren downstream. There are several sections of pipe that are close to the resonant length of 1180 mm, such as the water cooling loop.

The spatial distribution of the acoustical energy of the siren with the matching horn was measured. (That is, the siren without reflectors, or a collimating ring, but including the approximately exponential impedance matching horn.)

Figure 3 shows the dependence of the SPL on the horizontal distance from the center of the siren at several constant heights above the opening of the horn. The siren was operated at a frequency of 3260 Hz, at which the radius of the siren hole circle was 0.725 wavelengths. The most striking features of these plots is the regular progression of the minima, which may be expected to closely resemble the pattern of an ideal ring source. When the spatial locations of these minima are plotted in Fig. 4, they are seen to lie along a line which makes an angle of  $29^\circ$  with respect to the axis of the siren and extrapolates to the axis 45.4 mm below the outlet of the horn. (The siren holes are located 67.6 mm below the outlet of the horn, on a radius of 76.2 mm, as indicated in Fig. 4.) Allen and Rudnick (1947) observed the minima at approximately  $38^\circ$  at a radial distance of 25 cm from a similar siren operated at 3250 Hz

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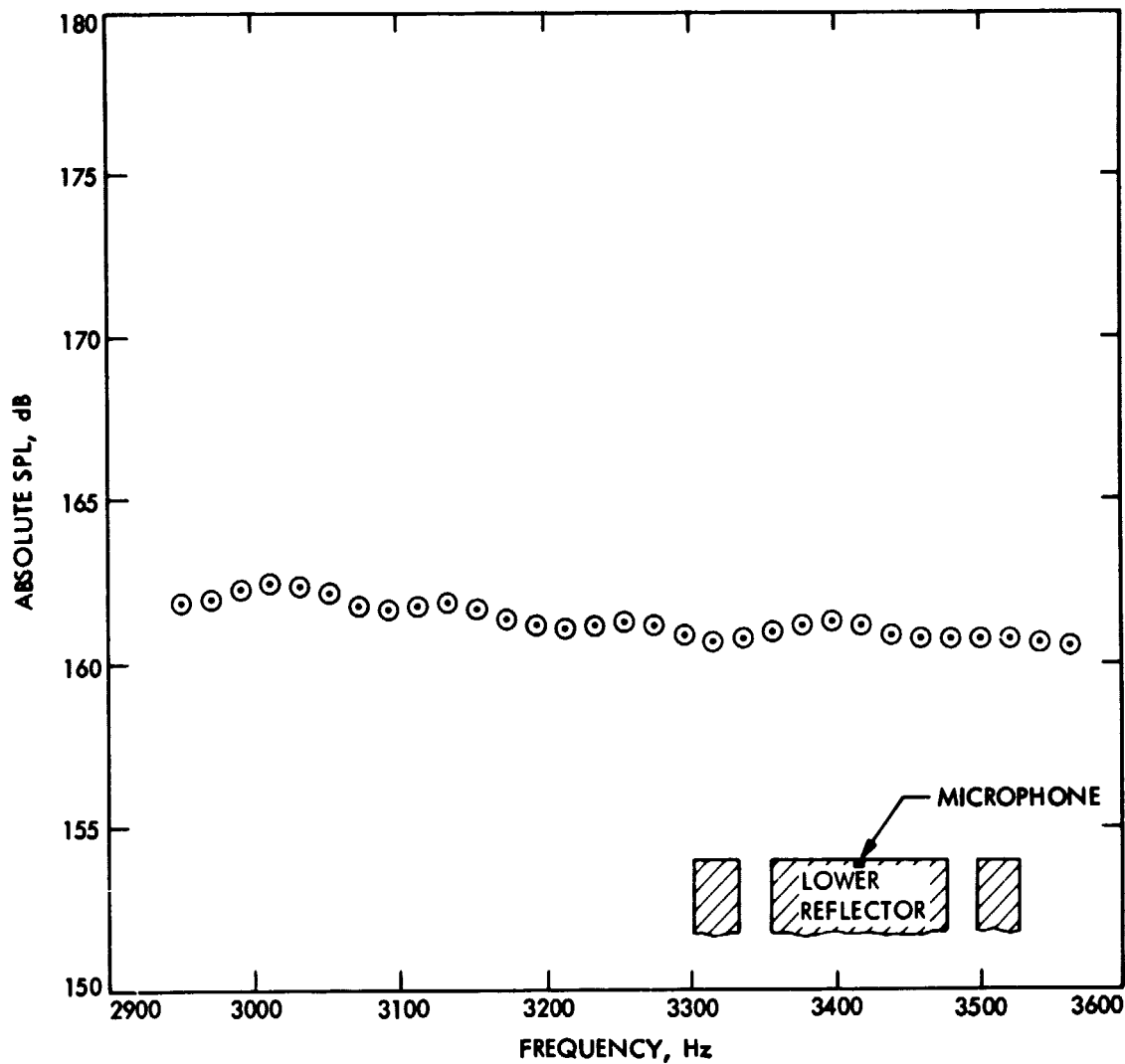


Figure 2. Acoustic output of the siren vs frequency. The microphone is located within the lower reflector, which is mounted atop the impedance matching horn. No upper reflector is present.

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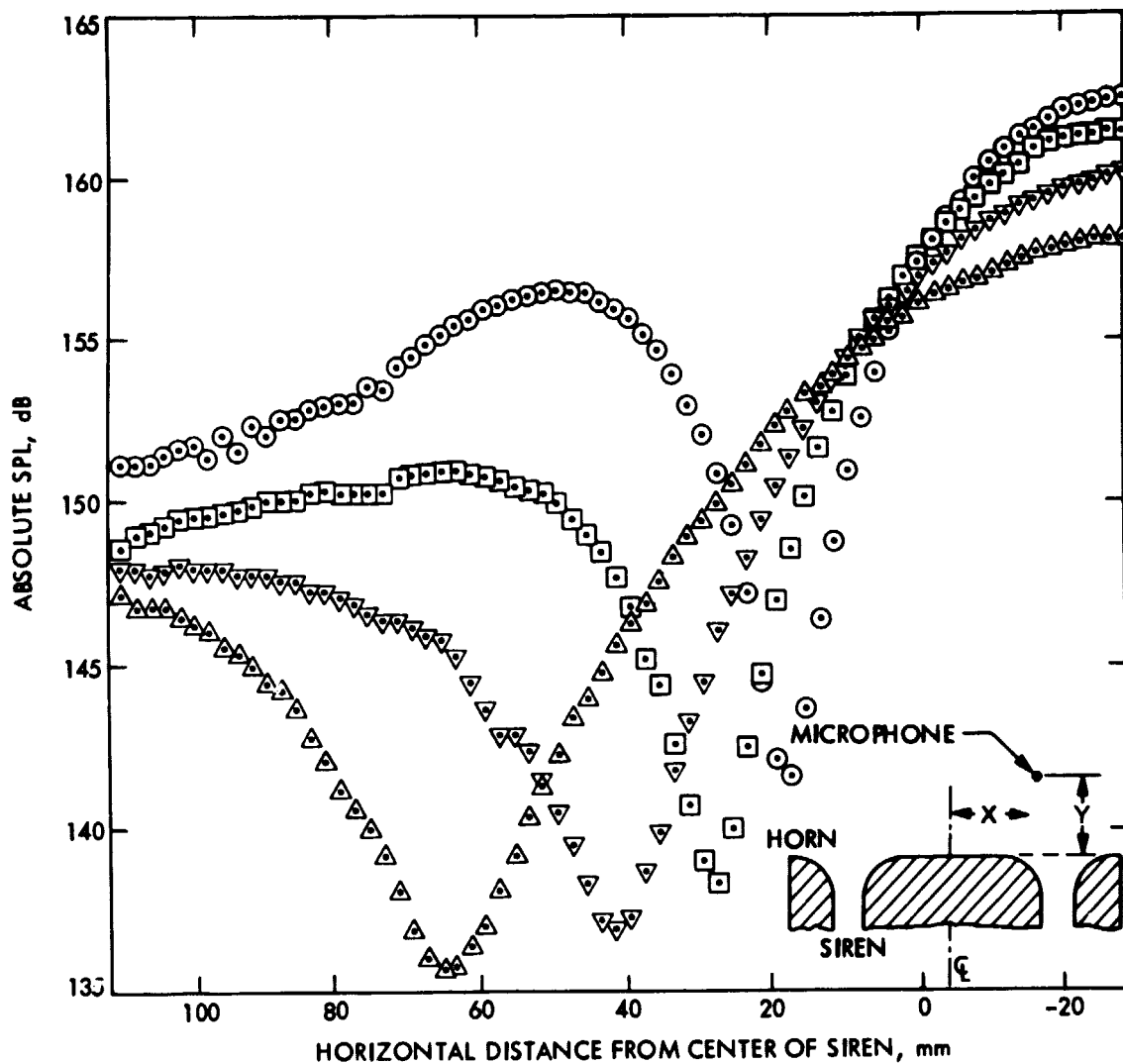


Figure 3. Acoustic output of the siren vs horizontal position at various heights above the horn opening. Reflectors are removed. The frequency is 3260 Hz.

- 25.4 mm above horn opening
- 50.8 mm above horn opening
- ▽ 76.2 mm above horn opening
- △ 101.6 mm above horn opening.

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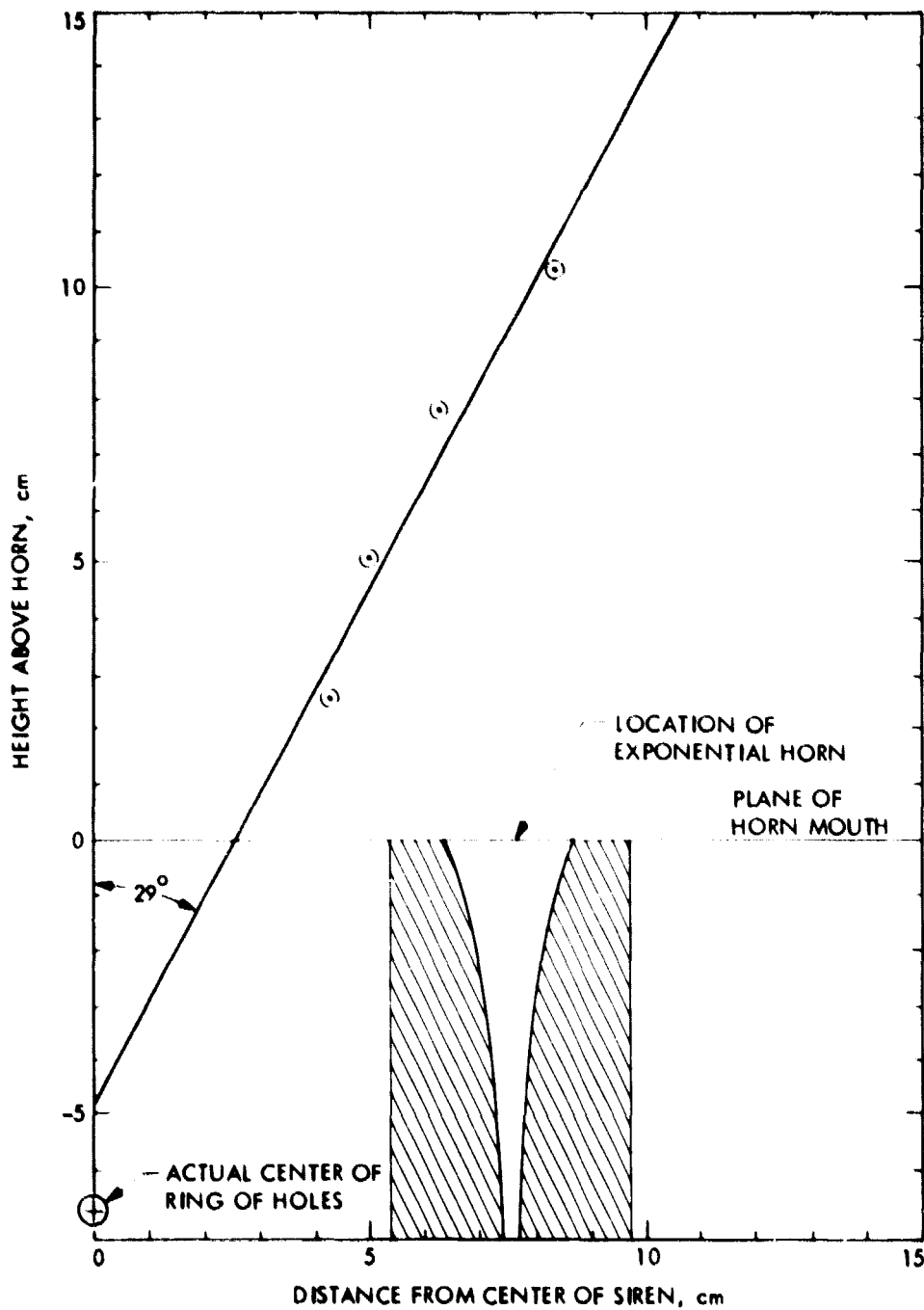


Figure 4. Spatial location of minima on acoustic field

with an exponential horn. When the present data is extrapolated to a 25 cm radius, the angle of this null with respect to the top of the horn, which is the reference plane used by Allen and Rudnick, is  $33.6^\circ$ .

That the apparent source point is not precisely at either the plane of the mouth of the horn or at the plane of the siren rotor disk is not at all surprising. Any impedance transformation device such as the exponential horn is expected to shift the virtual source point from which the sound appears to emanate. Heyser (1971) has reported that the apparent position of the source of a loudspeaker is behind its physical location for a loudspeaker with an imperfect frequency response.

The power levels are also similar, being 158 dB at 10 cm above the center of our siren and 155 dB 25 cm above the center of that used by Allen and Rudnick.

2. Siren With Reflectors. When the siren is used for acoustic levitation, a resonant reflector system is needed both to further increase the power and to produce the standing waves which provide an equilibrium levitation position. Many reflector geometries were investigated, including totally enclosed resonant chambers and pairs of reflectors that are open on the sides.

Both a cylindrical and a rectangular enclosed geometry were tried. With both of these, the sound was introduced through a hole in the center of one of the planar walls after being concentrated from the siren's ring source by a cone section. Both of these enclosed geometries resulted in very poor stability of the sample position. Several reasons are suspected for the instability. One is that the sound is concentrated to excessive levels, which causes the production of harmonics, especially the second harmonic. Another reason is that tightly confining the streaming and other convective currents results in their competing with the radiation pressure forces in determining the position of the sample (Lee and Feng, 1982).

Several open reflector geometries were tried: (1) two plane reflectors, (2) two concave reflectors, (3) a plane reflector on top and a concave reflector on the bottom, and (4) a concave reflector on the top and a flat reflector on the bottom. These geometries were investigated with various spacings between the reflectors and with several choices of the radius of curvature. The stability of levitation was observed throughout the vicinity of the fundamental resonance. (Generally, the most stable levitation was obtained slightly below the resonance.) The three

geometries that gave the most stable results are shown in Fig. 5. They are, in order of increasing stability: (1) plane parallel reflectors (Fig. 5a), (2) a flat lower reflector with a concave upper one (Fig. 5b), and (3) the same as (2), but with a modification from being a section of a hemisphere (Fig. 5c). (The modification consists of filling in the center of the hemisphere slightly.) In all cases, the lower flat reflector was arbitrarily chosen as 122 mm, which is of the order of a wavelength at the frequency used, namely around 3200 Hz.

Experiments were conducted on the effect of the reflector spacing and geometry on the power and stability of the levitation. In all cases the siren frequency was adjusted for optimum performance. Several upper concave reflectors were tried, with different radii of curvature. In each case, several reflector spacings were also tried and the siren frequency was varied at each to optimize the strength and stability of the levitation. A spacing of 66 mm between the 122 mm diameter plane reflectors of Fig. 5c was found to be optimal. A good choice of dimensions for the concave reflectors was a small (59 mm chord) portion of a hemisphere of 38.1 mm radius. The dimensions of these reflectors are in general agreement with the observation of Lee and Feng (1982) that the optimum reflector size is of the order of  $2/3$  of a wavelength.

Intermediate geometries (see Fig. 6), wherein a larger section of a hemisphere was used or the flat sides around the concave region were larger, produced poorer results in that the sample moved sideways more erratically. It was found that a full hemisphere of 38.1 mm radius gave good stability of levitation but could not lift steel balls even as small as 12.7 mm diameter. The small section of a hemisphere shown in Fig. 5b gave better stability and could repeatably levitate a single 19 mm diameter steel ball or three 12.7 mm diameter steel balls. A slight change from the geometry of Fig. 5c may produce an even stronger and/or more stable levitation.

## B. Observations of Sample Levitation

The system with a concave upper reflector repeatably and stably levitated steel and lucite balls of 12.7 mm and 19.05 mm diameters respectively. Fig. 7a demonstrates the levitation of a 19.05 mm diameter steel ball, while Fig. 7b demonstrates the stable levitation of an irregularly shaped glass object of a similar size. The natural tendency of objects to spin due to torques of the acoustic field (Busse and Wang, 1981) typically enhances the stability of a highly symmetric object. In the acoustic field of the siren-reflector system, the levitated object will usually spin in the horizontal plane. With a slight

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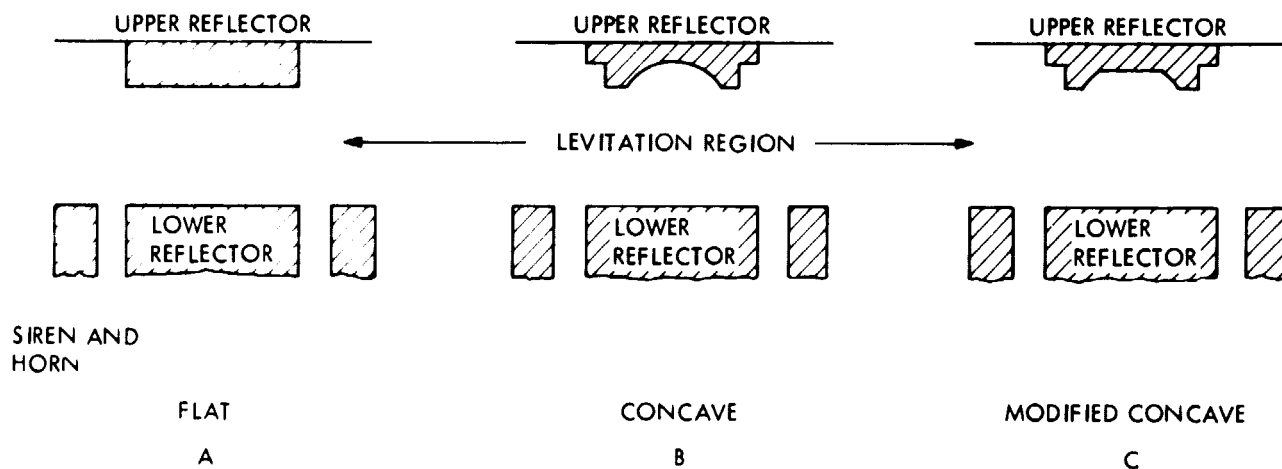


Figure 5. Reflector geometries studied

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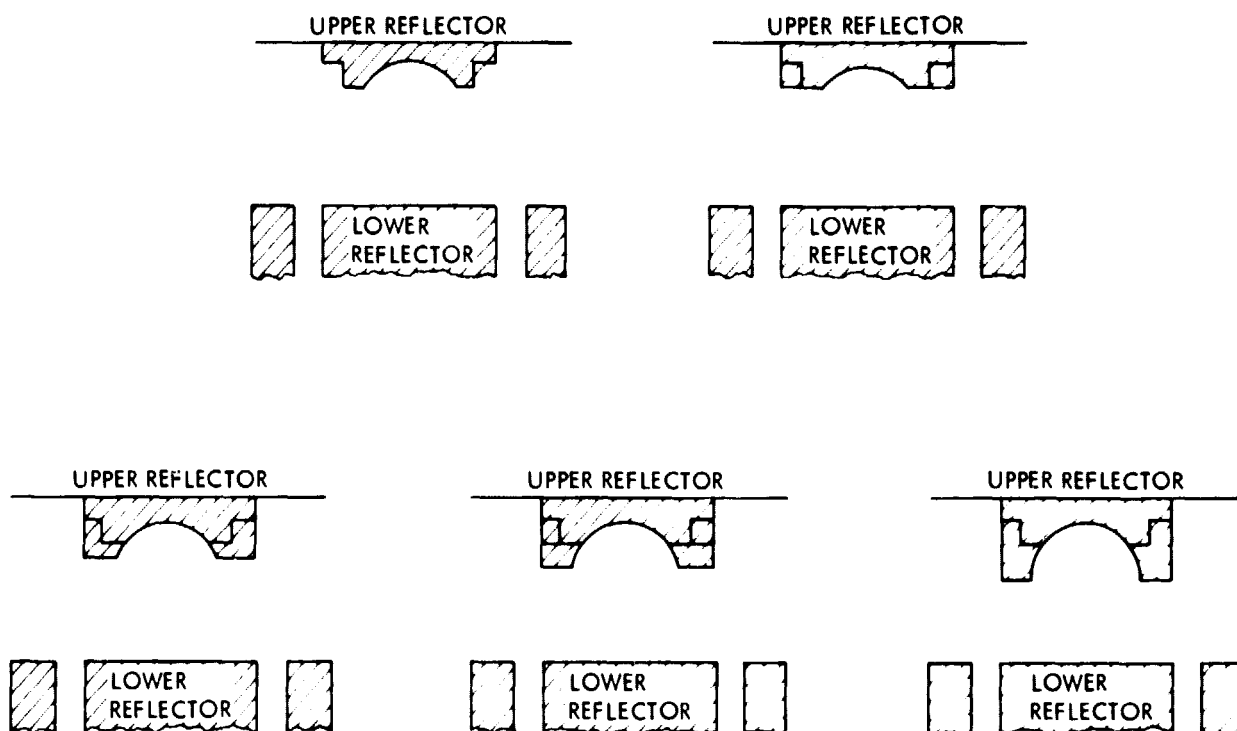


Figure 6. Intermediate concave reflector geometries



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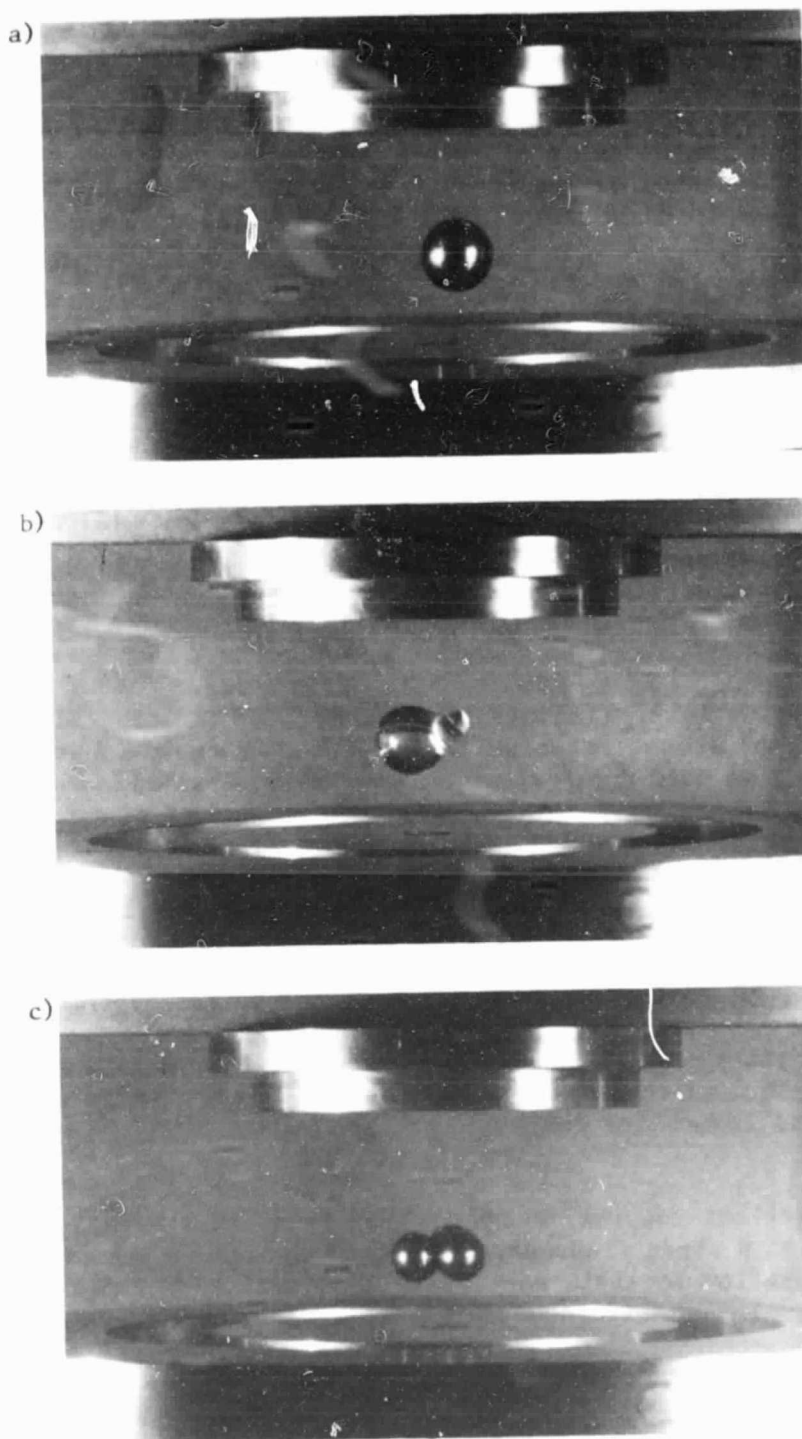


Figure 7. Dense objects being levitated by siren

- a) 19.05 mm diameter steel ball
- b) irregularly shaped glass object
- c) three steel balls

change of frequency or perturbation of the field, as by another object, spinning of the levitated object may slow down, reverse direction, then speed up in the opposite direction. An irregularly shaped object, such as shown in Fig. 7b, can be prevented from spinning by azimuthally perturbing the field, as by a small C-clamp attached to the edge of the upper reflector. The mutual attraction of three steel balls being levitated is demonstrated in Fig. 7c.

Generally the samples are levitated more stably when the siren frequency is slightly below the resonance of the reflector system.

Experiments were conducted to quantify the motion of the sample. A 15.875 mm diameter steel ball was coated with fluorescent paint and a 30 sec. time exposure taken of its motion using an ultraviolet light source. The reflector system consisted of a concave upper reflector and a flat lower one.

At the lower limit of levitation (3208 Hz), the sample motion was almost imperceptible. At the frequency that would normally be used for levitation with this particular reflector spacing (3232 Hz), a slight horizontal motion of approximately 4 mm and even less vertical motion was observed. Further increase of the frequency to 3242 Hz caused the sample to move in a downward facing arc (i.e., an inverted "U"). The limits of displacement of the sample were 2.5 cm horizontally and 1.0 cm vertically. This is the type of instability most commonly observed both with a flat and a concave upper reflector. If the hopping becomes extreme, as it would with some choices of reflector geometry, sample, and siren frequency, the sample may oscillate with ever increasing amplitude and eventually be ejected from the levitation region.

Vertical hopping, which is less frequently seen, was observed at a siren frequency of 3258 Hz, which is the upper limit of levitation for this particular combination of reflectors and sample. The amplitude of this hopping was 1.2 cm.

Occasionally the sample may move in a pendulum-like manner, describing a U-shaped path. This was observed with a 9.525 mm diameter steel ball in the same reflector geometry, at a frequency of 3218 Hz, 10 Hz above the lower limit of levitation, where the sample had been very stable. The vertical motion was 4 mm and the horizontal motion 15 mm.

## C. Measurements of the Sound Field

1. Comparison of Geometries. Measurements were made on the sound field within the levitation region and at the center of the lower reflector on all three geometries of Fig. 5 in order to determine the reasons for the greater levitation power and improved stability of Fig. 5c.

These measurements included measuring the sound pressure level, waveforms, and spectrum at the center of the lower reflector as the frequency was varied and mapping the pressure level in the levitation region. The effects of the presence of a sample and its position were also studied, using a 12.7 mm diameter steel ball attached to a small support rod, which will be referred to as a "restrained sample".

Fig. 8 compares the frequency response of the flat geometry of Fig. 5a with the geometry of Fig. 5b, which consists of a flat lower reflector and a concave upper one. This data was obtained with a 6.3 mm diameter microphone which was an integral part of the reflector and was flush with the face of the reflector at its center. The concave reflector has a higher Q, of about 54 (measured from the -3 dB points), as opposed to the Q of 29 for the flat reflector, and has 4 dB higher SPL at resonance, which accounts for its greater levitating ability. Data was also taken to compare the frequency response of the concave reflector system and the modified concave system (see Fig. 5b, 5c). The modification affects the magnitude of the resonance by less than 0.5 dB. The resonance was shifted up by less than 1% and the second and third harmonic content changed by 3 dB or less.

2. Effect of the Sample on the Frequency Response. The shift of the resonance with the vertical position of the sample can be predicted from excluded volume considerations (Smith et al., 1974). The dependence of the resonance on the horizontal position, which is more complicated, has recently been investigated by Barmatz, et al. (1982) for enclosed geometries. It is not clear that these results apply accurately to this open system, so the resonance shift was investigated empirically.

The curved reflector with perturbation increased the SPL at the lower reflector by 18 dB over that of the free field case. Putting the sample in the center of the region in which it would levitate lowered the resonance by 20 Hz but did not affect its magnitude appreciably. Moving the sample 13 mm to the side increased the resonance by about 5 Hz.

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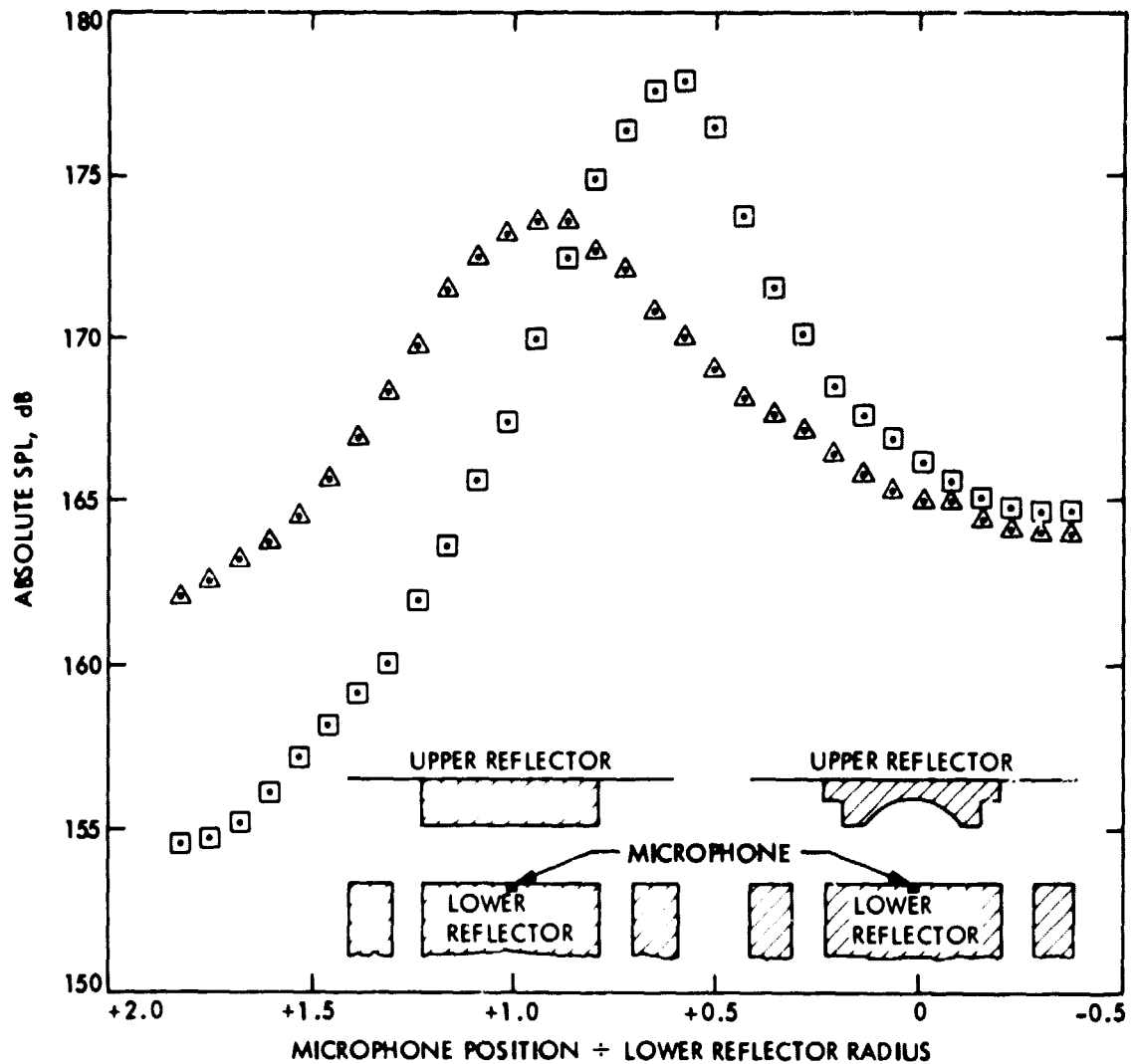


Figure 8. Comparison of frequency response with a flat and a concave upper reflector. Microphone is located flush to the surface of the flat lower reflector:

- △ flat upper reflector
- concave upper reflector.

The effect of the presence of the sample on the frequency response of the flat resonator of Fig. 5a and of the hemispherical geometry of Fig. 5b were compared. The introduction of the restrained sample reduced the Q of the resonance from 29 to 22.6 for the flat reflector geometry, but did not affect the Q of the concave geometry measurably. With both geometries, the presence of the restrained sample affected the amplitude at resonance by less than 0.5 dB. It shifted the resonance down, by 28 Hz for the flat reflector and by 20 Hz for the curved reflector.

The effect of change of sample position on the resonance was investigated. For the case of the flat upper reflector, moving the ball 25.4 mm horizontally from the center changed the amplitude of the SPL at the lower reflector by less than 1 dB and raised the resonant frequency by 20 Hz, while raising the Q from 22.6 to 26.6.

For the case of the concave upper reflector (Fig. 5b), the changes are almost immeasurable. The resonant frequency shifted by less than 5 Hz, the amplitude changed less than 0.5 dB, while the change in Q cannot be measured.

Perhaps one reason the hemispherical geometry is more stable is that its resonance is less sensitive to the sample position.

#### D. Relationship of Harmonic Content and Waveform

1. Without Reflectors. When the siren is operated without the upper reflector, but with the collimating rings, the waveform is triangular, as is evident in the oscilloscope trace (Fig. 9). Essentially the same waveform was observed for frequencies spanning more than an octave in the vicinity of the frequency used for levitation. All of the harmonics are at least 18 dB weaker than the fundamental, which contains over 98% of the energy. (For a symmetric triangular waveform the even harmonics are zero and the odd ones decrease at a rate of 12 dB/octave.)

2. With Flat Reflectors. The sound pressure field and the harmonic content of the flat reflector geometry of Fig. 5a were measured. A reflector spacing of 66 mm was used because this gave a resonant frequency close to that of the concave reflector system of Fig. 5c, with which comparisons are made.

Table 1 shows the harmonic content of the sound pressure waveform measured with a microphone that is an integral part of the face of the lower reflector, both with no sample present and with a restrained sample (12.7 mm diameter steel ball) at several locations representative of the normal motion of the levitated samples.

From the levels of the first five harmonics it is estimated that over 98% of the energy is in the fundamental for the cases studied in Table 1. The even harmonics of these waveforms are not zero, as they would be for a waveform with the appropriate symmetry. This lack of symmetry is a direct consequence of the pressure dependence of the compressibility of air at these intensities.

With no sample present, the strength of the second and third harmonics is decreased by 4 to 6 dB by using the lower, more stable frequency. The presence of a sample decreases these harmonics slightly, while its position has little effect.

The variation of the intensity of each of the harmonics with height above the lower reflector of the parallel reflector system can be predicted from simple resonance considerations. The parallel reflectors are spaced a half wavelength of the fundamental apart. All modes will exhibit a pressure maximum at the reflectors. The fundamental will exhibit a pressure null at the mid-plane and will be 3 dB down in the plane a quarter of the spacing above the lower reflector. All odd harmonics, in fact, have null intensity in the mid-plane. The second harmonic will be maximum at the mid-plane and null at one-fourth the spacing from either reflector.

Figure 10 shows the harmonic content and the corresponding waveforms of the sound field of the parallel reflector geometry for its resonance at the siren frequency of 3259 Hz at several locations in the sound field. Near the center of the field (30.5 mm above the bottom reflector, on the central axis), only 72% of the energy is in the fundamental and about 19% is in the second harmonic. The third and higher order harmonics each account for less than 1% of the energy. The second harmonic content is greatly increased over its value at the height of 15.5 mm above the reflector, where 97% of the energy is in the fundamental and only 2.5% is in the second harmonic.

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The increase of the harmonic content over that observed at the reflector (Table 1) is expected, since the microphone is closer to the mid-plane between the reflectors, where the fundamental and the odd harmonics theoretically cancel completely.

In Table 2 we see the absolute levels of the fundamental, the first, and the second harmonics as calculated from the measurements of the total SPL and the harmonic analysis.

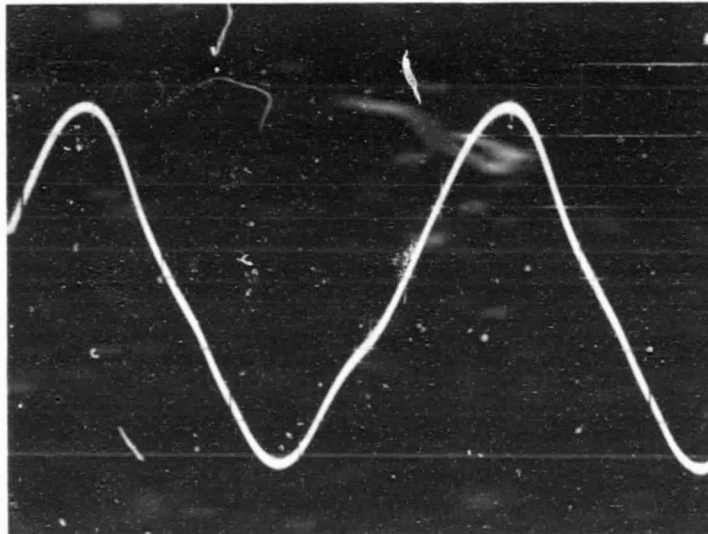
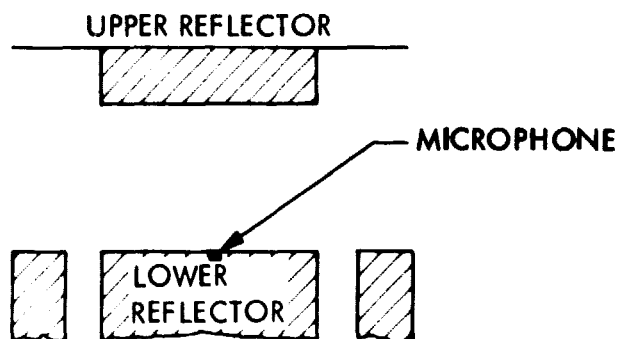


Figure 9. Sound pressure waveform at lower reflector, with upper reflector removed. This is representative of the free field waveform: from an oscilloscope trace.

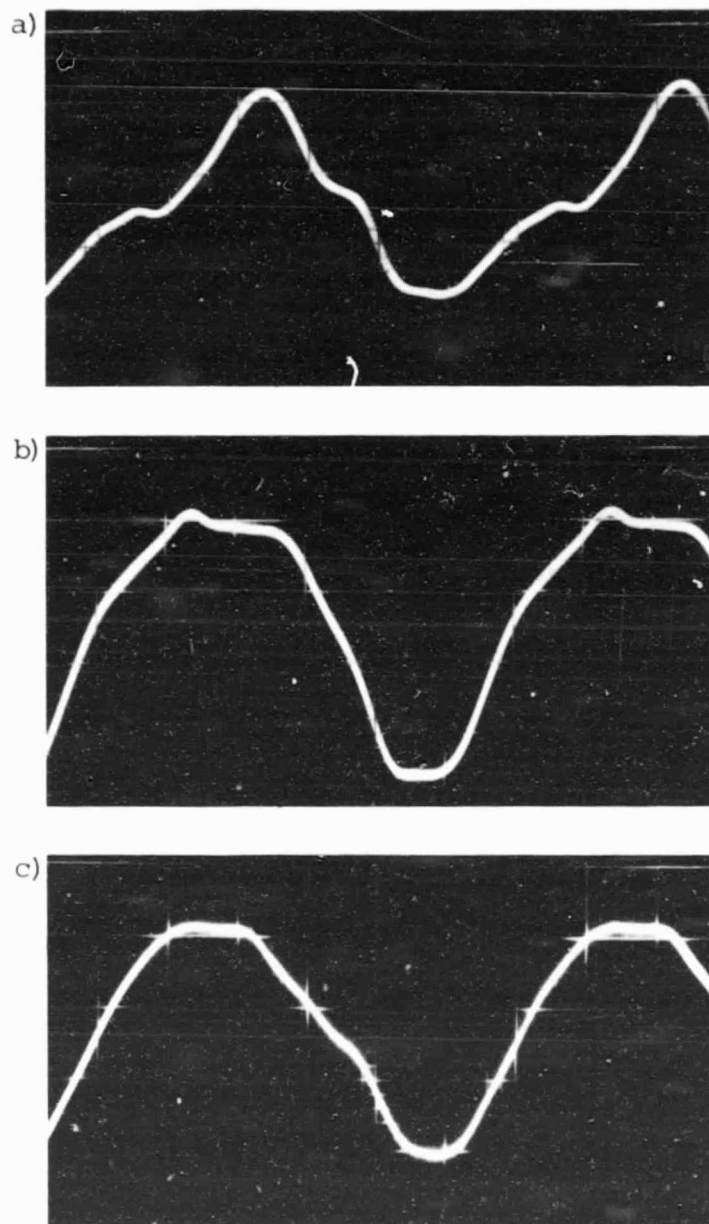
Table 1. Harmonic content of sound pressure at lower reflector,  
both reflectors flat

Conditions/Harmonic number	dB relative to fundamental			
	2nd	3rd	4th	5th
No sample, at resonance	24	24	29	30
No sample, below resonance	30	28	25	28
Sample in center, below resonance	32	23	26	30
Sample 12.7 mm from center, below resonance	32	24	27	30





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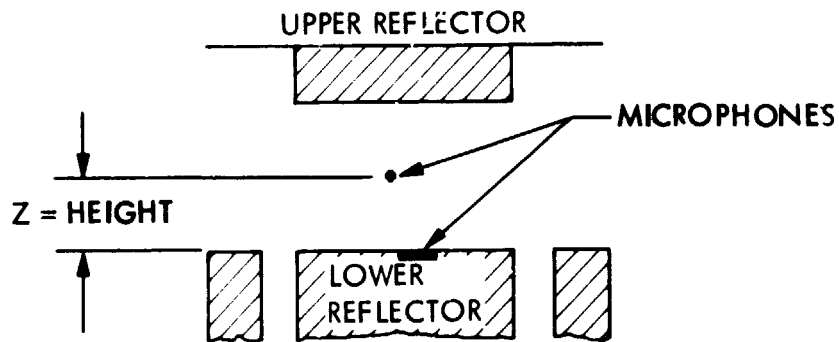
WAVEFORM	HEIGHT ABOVE LOWER REFLECTOR, MILLIMETERS	HARMONIC CONTENT, dB BELOW FUNDAMENTAL			
		2nd	3rd	4th	5th
a)	30.5	6	20	24	25
b)	22.9	9	27	31	36
c)	15.2	16	28	32	35

Figure 10. Waveforms and harmonic content for flat reflector geometry

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Table 2. Harmonic content vs height from lower reflector,  
both reflectors flat

	Height above lower reflector, mm		
	0.0	15.2	30.4
Total SPL, dB	173	171	164
Power in fundamental, %	99	97	73
SPL of fundamental, dB	173	171	163
Power in second harmonic, %	0.4	25	20
SPL of 2nd harmonic, dB	149	155	157
Power in third harmonic, %	0.4	0.2	0.8
SPL of 3rd harmonic, dB	149	144	143



We note that towards the middle of the field the fundamental has decreased a full 10 dB from the reflector while the second harmonic has increased by 8 dB. (The third harmonic has fallen only by 6 dB.) This would be expected as the fundamental and third harmonic have a pressure nodal plane mid-way between the reflectors, whereas the second harmonic is maximum there. For this reason it may well be that the second harmonic is the principal destabilizing influence in the plane reflector geometry. The second harmonic is only 7 dB below the fundamental. Since the theory of King (1934) predicts the force to increase as the square of the pressure and inversely with the wavelength, the force due to the second harmonic must be 0.89 times that of the fundamental. Note that the second harmonic magnitude (157 dB) is greater than that used to levitate heavy objects in zero gravity and light samples (e.g., styrofoam balls) in 1-G in other systems. It is no wonder that a slight change in the second harmonic can seriously affect lateral stability in this system.

Quite likely the reason that operation below resonance is more stable is that the second harmonic content is reduced. This also serves to keep the sample out of the mid-plane, which is where the second harmonic predominates. It has been repeatedly observed that the most stable levitation is when the sample is barely levitated, i.e., when it is only levitated slightly above the plane of maximum force, an eighth of a wavelength above the lower reflector.

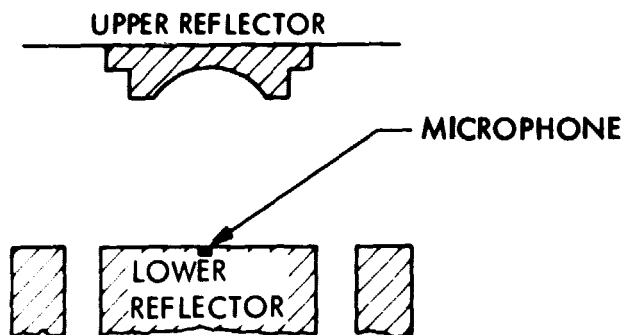
3. With Concave Upper Reflector. The harmonic analysis of the pressure waveforms measured by a microphone that was an integral part of the lower reflector is shown in Table 3. With the curved upper reflector in place the harmonic content varies considerably with frequency. The frequency of 3216 Hz gives the most stable levitation, while the frequency of 3298 Hz lifts the sample higher, but with less positional stability. With the lower frequency all harmonics are at least 15 dB below the fundamental, which contains 95% of the total energy. This is in contrast with the higher frequency, where the second harmonic is only 4 dB below the fundamental, which contains only 69% of the total energy. Modes other than the fundamental undoubtedly contribute to the instability, since the pressure distribution and thereby the force distribution is complicated by these higher order modes.

The results with an unperturbed upper reflector are very close to those with the perturbation. The presence of the sample seems to lessen the effect of the frequency on the sound pressure measured at the lower reflector. For example, at the frequency above that of stable levitation, the second, third, and fourth harmonics are decreased when the sample is in the center, whereas

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Table 3. Harmonic content of sound pressure at face of flat lower reflector for concave upper reflector

Conditions/Harmonic number	dB relative to fundamental			
	2nd	3rd	4th	5th
No reflector, below resonance	21	18	30	>37
Reflectors, below resonance	18	15	40	26
Reflectors, at resonance	4	16	18	26



at the frequency of the most stable levitation the sample being at the center increased these harmonics. (The third harmonic is practically unchanged, however.) Moving the sample 12.7 mm from the center changes the harmonic content slightly.

#### E. Mapping the Sound Field

In order to better understand the factors that affect the mass of a sample that can be levitated by the siren-reflector system and its positional stability, the sound pressure field was mapped for the three reflector geometries shown in Fig. 5. The general variation of the sound field intensity and the pressure waveforms at different locations in the preferred configuration of the siren-reflector system can be seen in Fig. 11. The upper reflector is hemispherical and the lower reflector flat. More detailed mapping and spectral analysis are presented in the following figures. In the plots that follow, the absolute sound pressure level is plotted against the microphone position. The horizontal coordinate of the microphone position is expressed in units that are normalized by the size of the lower reflector, namely the distance of the microphone from the center of the reflectors divided by the radius of the reflectors. Traverses were made with the microphone at varying height above the lower reflector, which was a parameter of these families of plots.

Figure 12 shows the distribution of the sound pressure for the flat reflector geometry of Fig. 5a. Figure 12a shows the sound pressure at points in the central plane (midway between the reflectors) and below that plane. This is the region in which the levitated sample normally resides. Figure 12b shows the sound pressure at points above the central plane.

Figure 13 shows the mapping of the sound pressure field for the reflector geometry of Figure 5c, where the lower reflector is flat and the upper reflector is a section of a hemisphere, modified by filling in the central portion slightly. A free sample levitates to a maximum height of approximately 25.4 mm above the lower reflector.

In order to rule out the possibility that the flatness of the curve for the 12.7 mm elevation was due to the sound field saturating near 175 dB, the experiment was repeated, varying the pressure head of the air supply to the siren. The field plot retained its shape even when the siren output was reduced 13 dB

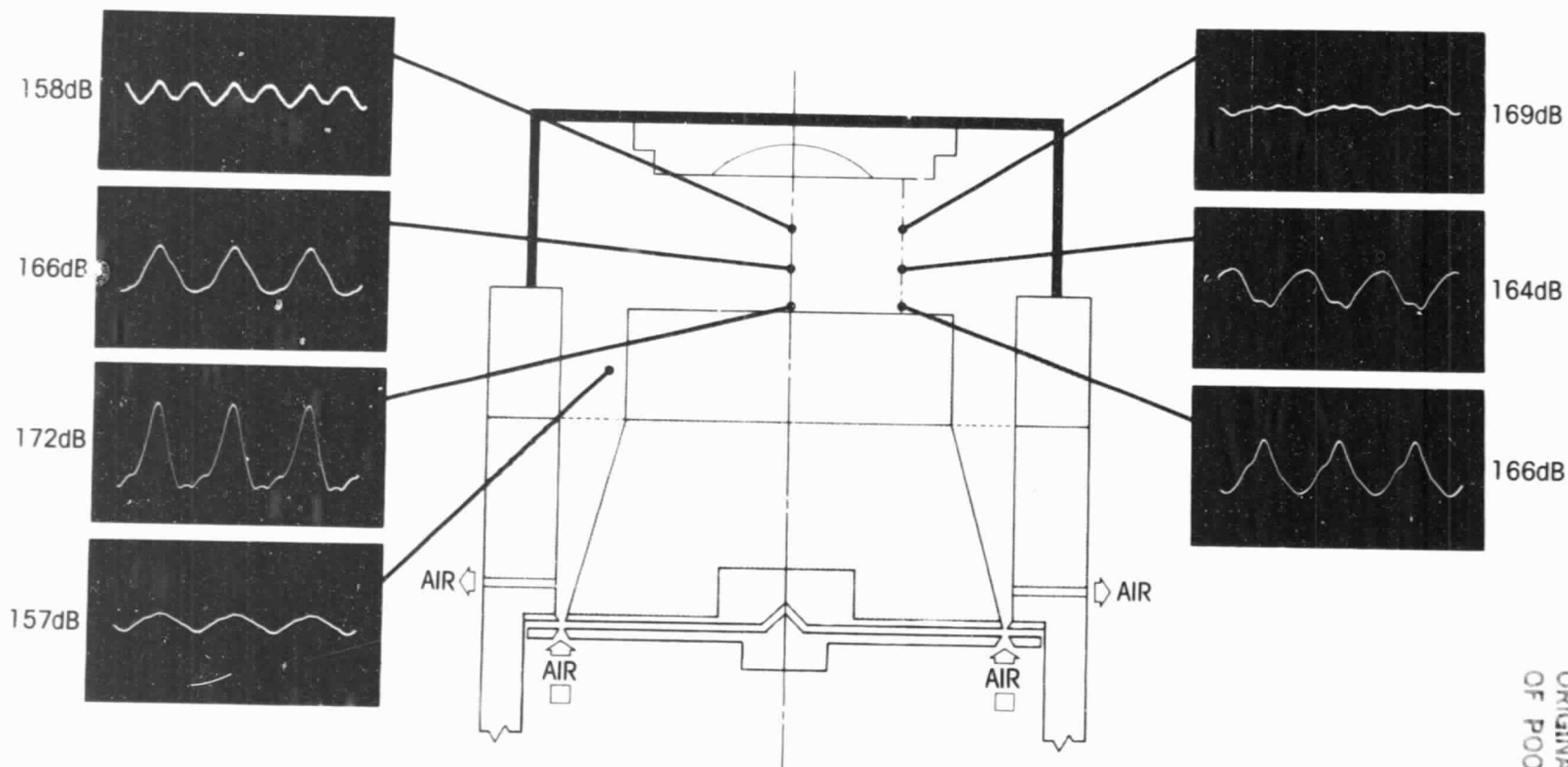


Figure 11. Acoustic pressure waveforms in the siren-reflector system

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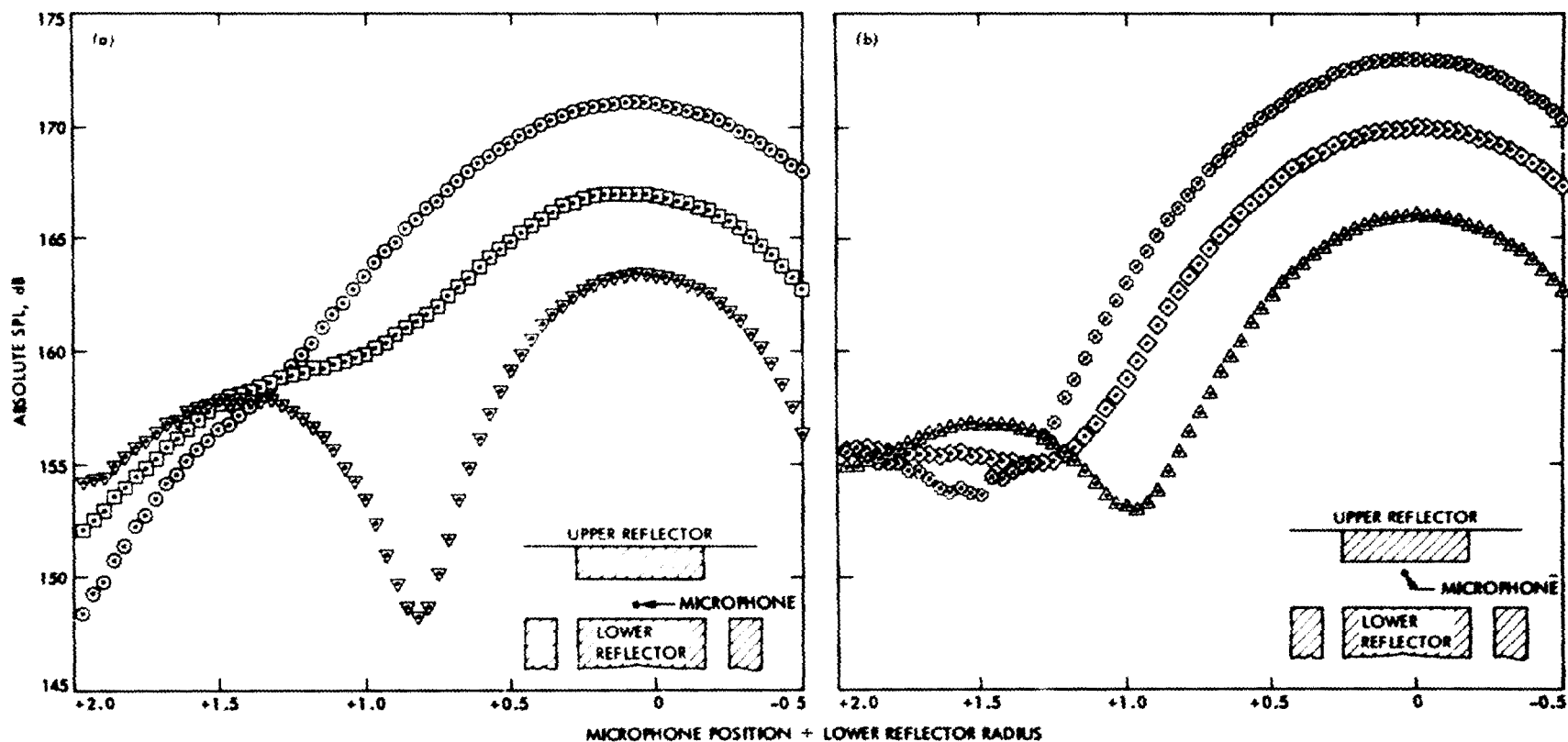


Figure 12. Acoustic field mapping for flat upper and lower reflectors. Horizontal plots shown for various heights above the lower reflector: (a) at heights below the central plane, (b) at heights above the central plane:

- 15.2 mm above the lower reflector
- 22.9 mm above the lower reflector
- ▽ 30.5 mm above the lower reflector

- △ 38.1 mm above the lower reflector
- ◇ 45.7 mm above the lower reflector
- ⊙ 53.3 mm above the lower reflector

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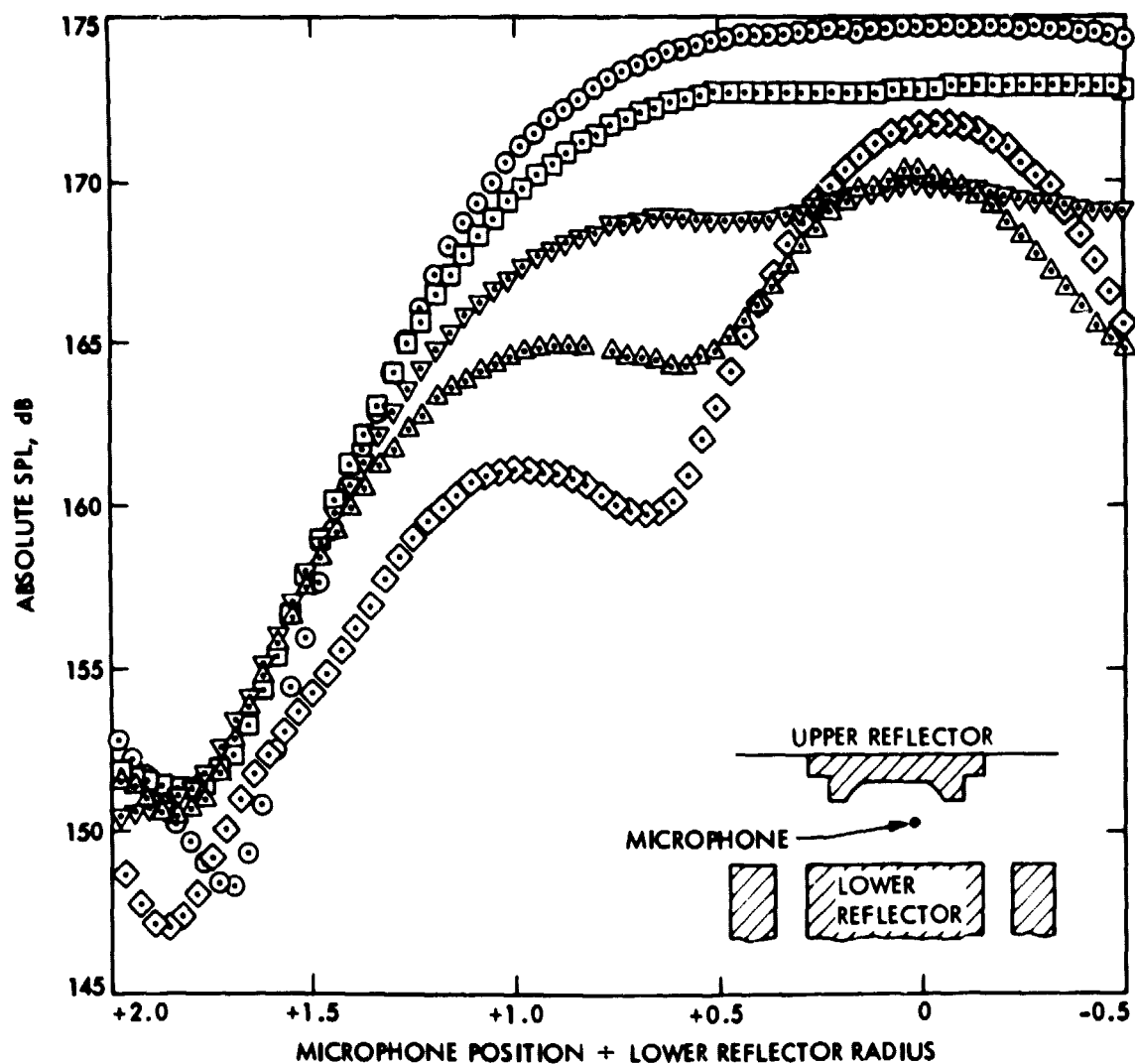


Figure 13. Acoustic field mapping for flat lower reflector and modified concave upper reflector. Horizontal plots shown for various heights above the lower reflector:

- ⊙ 12.7 mm above the lower reflector
- 19.1 mm above the lower reflector
- ▽ 25.4 mm above the lower reflector
- △ 31.8 mm above the lower reflector
- ◇ 38.1 mm above the lower reflector



by decreasing the pressure of the air supply from around 7.5 psi to 3 psi.

In Fig. 14 the sound pressure plots of the reflector geometries which use a flat upper reflector and which use a modified concave upper reflector are compared for the region between the lower reflector and the central plane where a freely levitated sample would normally be. Data is shown both near the plane of maximal force (i.e., 12.7 mm and 15.24 mm above the lower reflector for the flat and curved geometry, respectively) and near the plane of zero force (19.05 mm and 22.86 mm for the respective reflectors). The curved reflector geometry is seen to give a stronger and generally more uniform field in the region.

In Fig. 15 the SPL of the curved reflector geometry with the upper reflector not modified (i.e., the reflector is a section of a perfect sphere) is shown. This can be compared with the modified concave geometry of Fig. 13. The modification has made the  $Z = 12.7$  mm plot of the curved reflector flatter and of higher level by about 2 dB. For  $Z$  greater than 25.4 mm, where the sample will never be, the intensity distribution of both the modified and the unmodified concave upper reflectors are more peaked than for the flat upper reflector.

The modification was seen to make very slight ( $\sim 1$  dB) change in the harmonic content measured 12.7 mm above the face of the lower reflector, and imperceptible change in the waveform. This was observed both in the center of the field and 12.7 mm from it.

#### F. Effect of the Sample and Its Position on the Spatial Distribution of the Pressure Field

It has been observed by other investigators and confirmed by the present study that a mutual attraction exists amongst objects that are levitated in the same sound field. This is demonstrated in Fig. 7c. Levitated objects will be attracted to any perturbation, such as a rod, that is moved into the field, and can be led about in the field by such an object. This is because of the increase in the sound pressure upon reflection from the solid object. The sample itself causes such a localized increase in sound pressure, which undoubtedly plays an important part in the positional stability of the sample. For this reason, it is highly desirable to determine the effect of the sample on the sound field.

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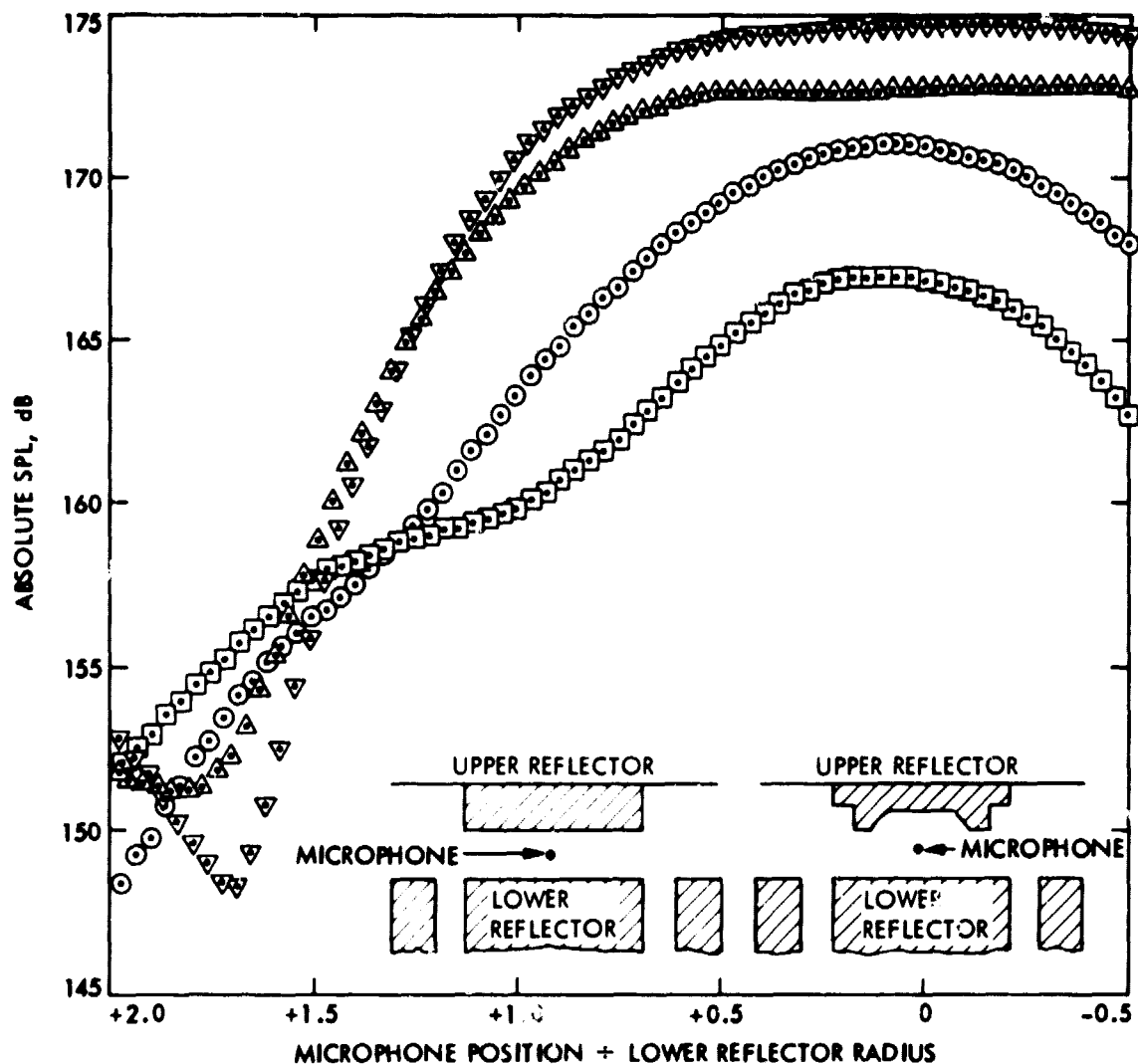


Figure 14. Comparison of the acoustic field in the levitation region produced by the flat and the modified concave reflector geometries. Lower reflector is flat.

- flat upper reflector, 15.2 mm above the lower reflector
- flat upper reflector, 22.9 mm above the lower reflector
- ▽ modified concave upper reflector, 12.7 mm above the lower reflector
- △ modified concave upper reflector, 19.1 mm above the lower reflector

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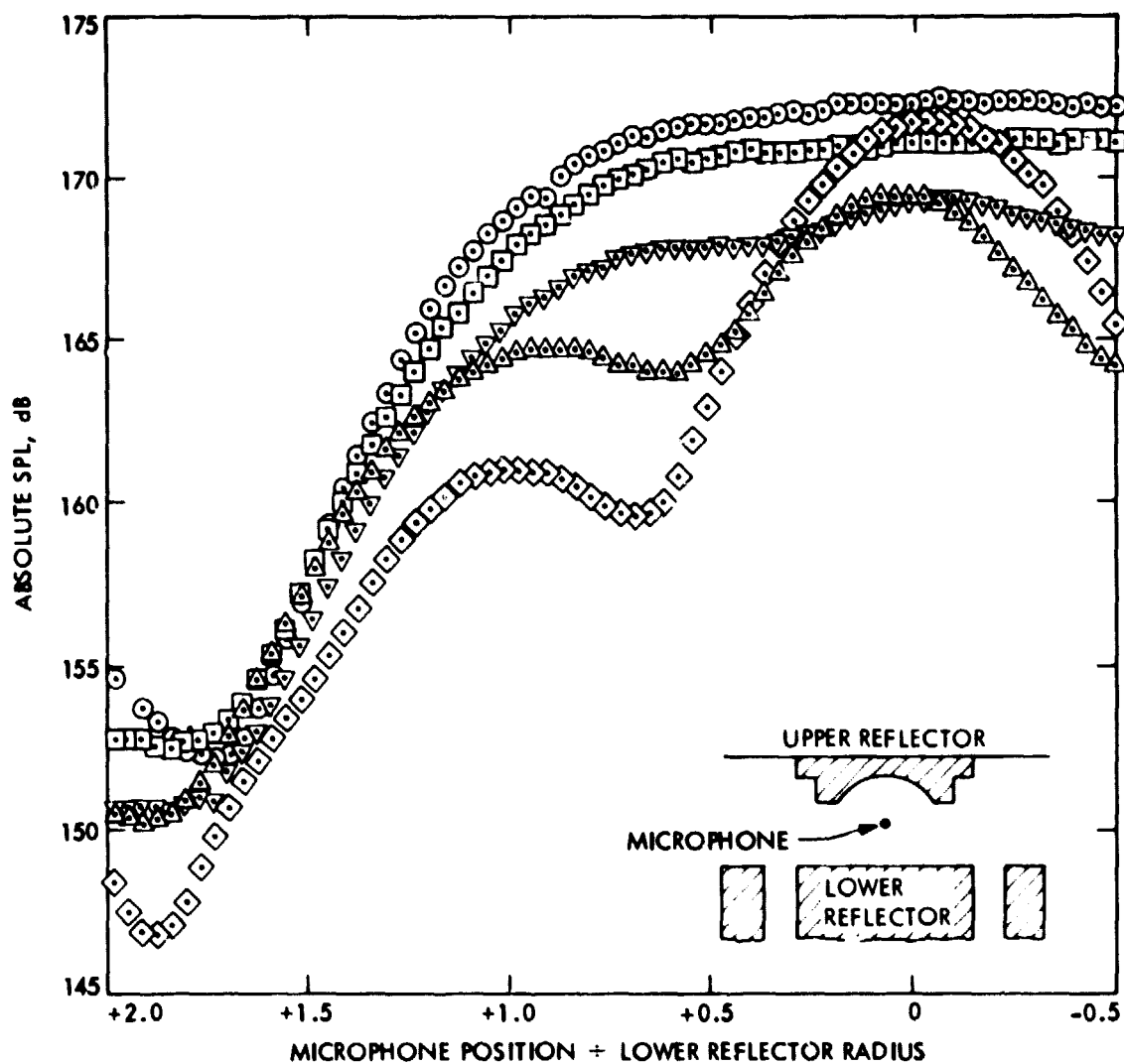


Figure 15. Acoustic field mapping for flat lower reflector and spherically concave upper reflector. Horizontal plots are shown for various heights above the lower reflector:

- $\odot$  12.7 mm above the lower reflector
- $\boxdot$  19.1 mm above the lower reflector
- $\nabla$  25.4 mm above the lower reflector
- $\triangle$  31.8 mm above the lower reflector
- $\diamond$  38.1 mm above the lower reflector

The perturbing effect of the sample on the sound field was simulated with a restrained sample. The sound pressure field was mapped with the small (6.3 mm diameter) microphone, mounted on a rod as described earlier.

Figure 16 demonstrates the change of the sound field with sample position when the frequency is slightly below the resonance of the parallel reflector resonant system of Fig. 5a. The frequency was fixed at 3270 Hz, which was the resonant frequency of the system, and the SPL was mapped both with the sample removed and with the sample at two locations: one in the center of the reflectors and the other 12.7 mm to one side. Although the effect of the sample is slight (approximately 1 dB), this data clearly demonstrates that the spatial maximum of the SPL follows the sample. The sample was a 12.7 mm diameter steel ball, restrained at a height of 30.5 mm above the flat bottom reflector at the designated horizontal position. The 6.3 mm diameter microphone was traversed horizontally at a height of 15.2 mm above the lower reflector.

Figure 17 demonstrates the effect of the sample position on the spatial distribution of the sound field for the modified curved reflector geometry of Fig. 5c. The upper reflector geometry is perturbed slightly from a section of a sphere. The frequency was 3216 Hz during these measurements, which was below the resonant frequency. The sample was a 12.7 mm diameter steel ball which was restrained at a height of 25.4 mm above the flat bottom reflector at the designated horizontal position. The 6.3 mm diameter microphone was traversed horizontally at a height of 15.2 mm above the lower reflector. The sample produces an increase of the SPL by about 2 dB in its immediate vicinity. This increase results in an attraction among objects that are levitated simultaneously. When this experiment was repeated with the upper reflector not modified as in Fig. 5b, the modification was seen to make little difference. The perturbation of the field by the sample probably increases the positional stability.

Although the modified concave reflector geometry gave excellent stability and could lift heavy samples at room temperatures, problems were encountered when localized heating was combined with levitation. Attempts were made at heating the sample with a light source, at electrically heating the acoustic reflectors, and with supplying heat from a hot air jet which was an integral part of the lower reflector. In all cases the acoustic field carried the heat away rapidly. Furthermore, the temperature gradients increased the stability problems. Oran (1979) suggests that localized heating and acoustic levitation may well be incompatible. The solution possibly lies in enclosing the levitation apparatus in an ovenlike arrangement.

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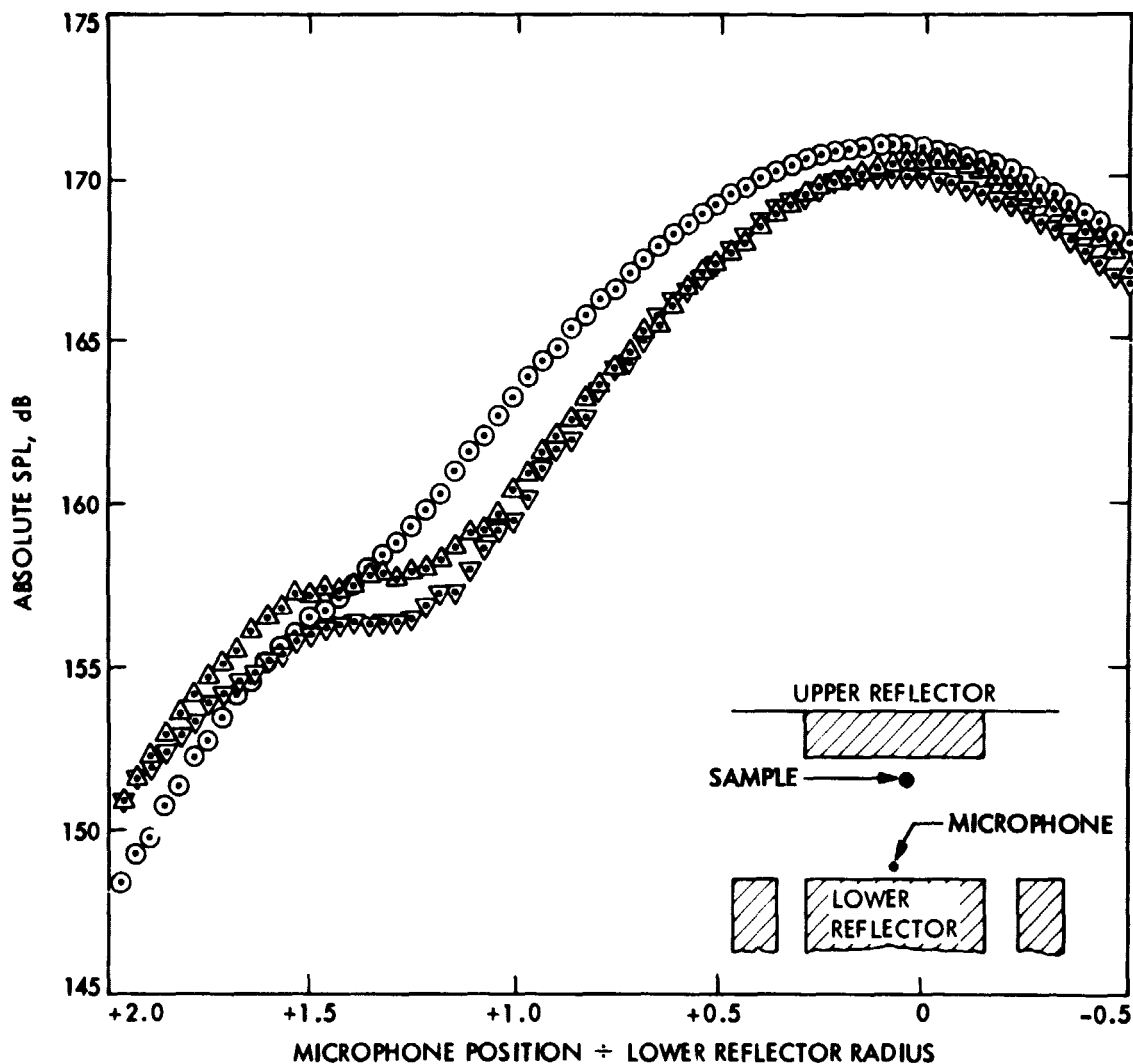


Figure 16. Effect of sample on sound field with flat upper and lower reflectors. Restrained sample is a 12.7 mm diameter steel ball located 30.5 mm above the lower reflector. Microphone traversed 15.2 mm above the lower reflector.

- sample removed
- ▽ sample on axis
- △ sample 12.7 mm from axis

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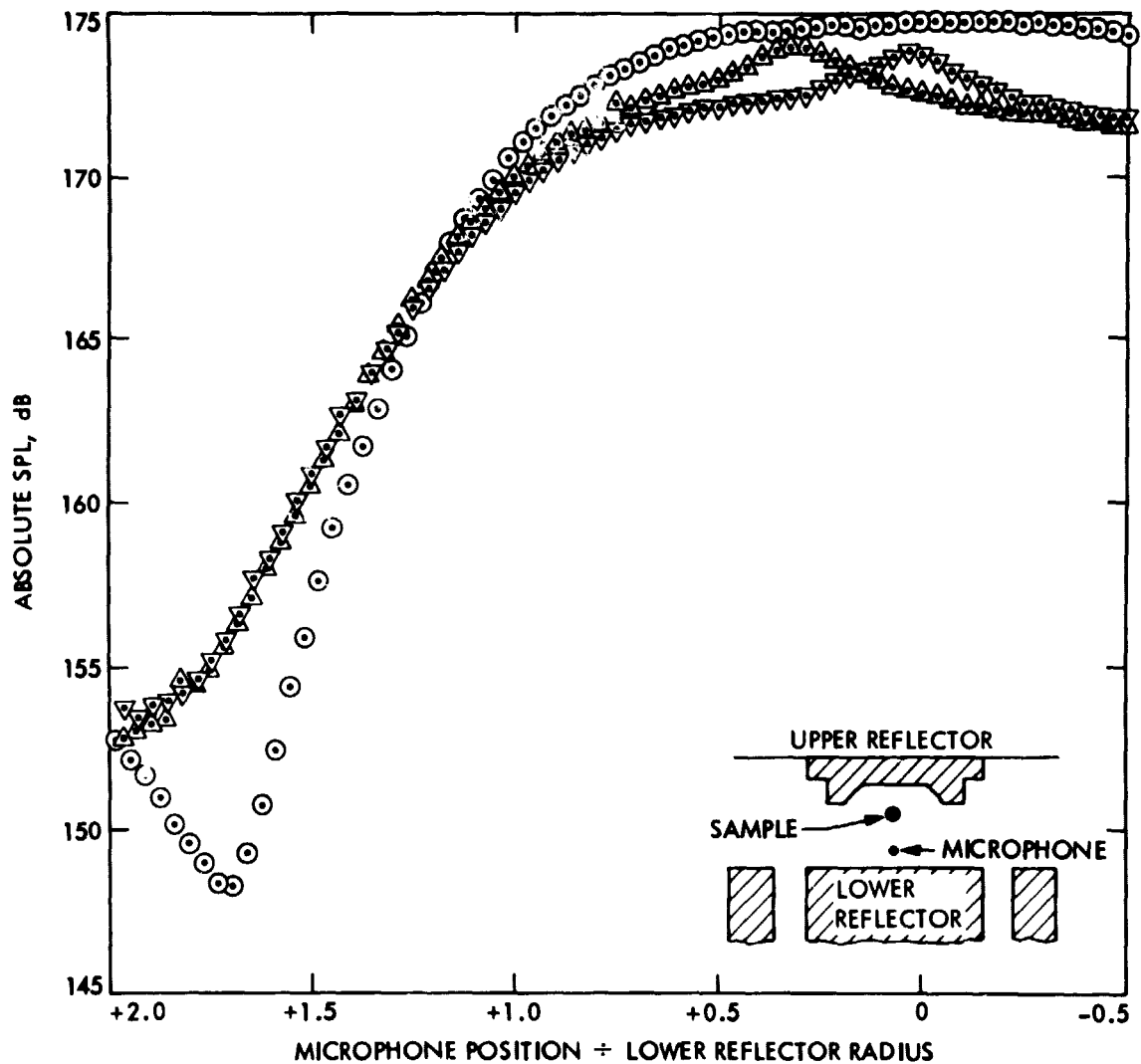


Figure 17. Effect of sample on sound field with flat lower reflector and modified concave upper reflector. Restrained sample is a 12.7 mm diameter steel ball located 25.4 mm above the lower reflector.

- sample removed
- ▽ sample on axis
- △ sample 12.7 mm from axis

## V. CONCLUSIONS

This study has demonstrated the feasibility of acoustically levitating high density objects of approximately 1 cm size in the earth's gravitational field, using the siren as an acoustic power source. Many factors affecting the stability of the levitation and other characteristics relevant to optimizing the system have been identified and considerable progress towards an optimized design has been made. Although further optimization is possible, this design is expected to be adequate for most containerless processing applications.

An impedance matching horn of some type is necessary for efficient transfer of energy from the siren to the levitation region.

The most stable levitation of the designs studied was obtained with a reflector system consisting of a flat lower reflector, close to the siren, and a concave upper reflector. Their spacing is approximately one-half a wavelength, while their lateral dimensions are of the order of  $2/3$  wavelength. Good results were obtained when the upper reflector was a small segment of a sphere, and even better results were obtained when it was perturbed slightly from being a perfect segment of a sphere.

The optimum operating frequency was found to be slightly below the actual resonance. Frequencies below the resonance were observed to result in a lower harmonic content. When multiple modes are excited, the situation is complicated and the second harmonic is expected to be a particularly strong destabilizing factor as it is maximal in the plane midway between the reflector, at the very place where the fundamental is null. The force of the second harmonic can conceivably be greater than 50% of the force of the fundamental. Use of acoustic power in excess of that needed to maintain the sample slightly above the plane of maximum force is counterproductive in two ways: 1) The second harmonic content is increased, and 2) the sample is moved closer to the mid-plane, where the fundamental sound pressure is null and the second harmonic is maximal. Thus, as power is increased, the behavior of the sample becomes increasingly dominated by the second harmonic. The well controlled frequency provided by a good motor speed control was essential for these studies.

The reflector geometries that were open on the sides performed well. Enclosed geometries result in extremely unstable operation, probably due to the confinement of the sonic wind (streaming).

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